SM 122 Quiz #7B (4 Feb 04: TSM) Spring 2004 Last Name: SOLUTION

Time: 6 minutes: Closed book, closed notes, no calculator allowed

1. Complete the statement of the formula for integration by parts.

$$\int u\,dv = uv - \int v\,du$$

2. Our proof in class (and in the book) of the integration by parts formula relied on:

Circle ALL correct answers

(a) product rule for derivatives $\checkmark$
(b) definition of definite integral
(c) Fundamental Theorem of Calculus $\checkmark$
(d) mean value theorem
(e) the chain rule

3. We may evaluate the integral

$$\int_0^1 2x\arctan(x)\,dx$$

using integration by parts with $v = x^2 + 1$.
This is an unusual choice for v, but it will work!
Fill in the three remaining blanks.

$$u = \arctan(x) \qquad dv = 2x\,dx$$

$$du = \frac{1}{x^2+1}\,dx \qquad v = x^2 + 1$$

4. Evaluate the integral in #3 using the scheme above.
(Give an exact answer involving π.)

$$\begin{aligned}\int_0^1 2x\arctan(x)\,dx &= \int_0^1 u\,dv = uv\big|_0^1 - \int_0^1 v\,du \\ &= (x^2+1)\arctan(x)\big|_0^1 - \int_0^1 (x^2+1)\cdot\left(\frac{1}{x^2+1}\right)dx \\ &= 2\arctan(1) - \arctan(0) - \int_0^1 1\,dx \\ &= 2\left(\frac{\pi}{4}\right) - 0 - 1 \\ &= \frac{\pi}{2} - 1\end{aligned}$$